A Durability Model Incorporating Safe Life Methodology and Damage Tolerance Approach to Assess First Inspection Period for Structures

J J Xiong\textsuperscript{1}, R A Shenoi\textsuperscript{2} *

\textsuperscript{1}Aircraft Department, Beihang University, Beijing, 100083, People’s Republic of China
\textsuperscript{2}, \textsuperscript{2}School of Engineering Sciences, University of Southampton, Southampton SO17 1BJ, UK.

(Corresponding author)

Abstract This paper outlines a new durability model to assess the most economical first inspection period for structures. Practical scatter factor formulae are presented to determine the safe fatigue crack initiation and propagation lives from the result of a single full-scale test of a complete structure. Theoretical prediction techniques are developed to establish the relationship equation between the safe fatigue crack initiation and propagation lives with a specific reliability level using a two-stage fatigue damage cumulative rule. A new durability mathematical model incorporating safe life and damage tolerance design approaches is derived to assess the most economical first inspection period. Finally, the proposed models are applied to assess the most economical first inspection period of a fastening structure at the root of helicopter blade.

Key words durability, safe life, damage tolerance, economical first inspection period, reliability level

NOTATION

\(a\) crack size
\(a_0\) visually detectable crack size
\(a_c\) critical crack length
\(A\) shape parameter of \(p - s_a - s_m - N\) surface
\(f\) fatigue crack opening function
$f(s)$ probability density function of cyclic stress of $s$

$f(s_a, s_m)$ probability density function of cyclic stress of $(s_a, s_m)$

$K_{IC}$ fracture toughness

$\Delta K$ stress intensity factor range

$\Delta K_{th}$ fracture threshold value

$n$ sample size

$n_i$ number of $i$th cyclic stress in a block of load spectrum

$n_f$ total frequency of cyclic stress in a block of load spectrum.

$n(s_a, s_m)$ number of cyclic stress of $(s_a, s_m)$

$N_i$ cycle number to fatigue crack initiation under the independent loading of $i$th cyclic stress

$N_{pi}$ cycle number to fatigue crack initiation under the independent loading of $i$th cyclic stress pertinent to a reliability level of $p$

$N_{p}(s)$ cycle number to fatigue crack initiation under the independent loading of cyclic stress of $s$ pertinent to a reliability level of $p$

$N_{p}(s_a, s_m)$ cycle number to fatigue crack initiation under the independent loading of cyclic stress of $(s_a, s_m)$ pertinent to a reliability level of $p$

$p$ reliability level

$r$ stress ratio

$r_s$ reduction factor of fatigue strength

$s$ standard deviation of the sample, fatigue stress

$s_a$ stress amplitude

$s_m$ mean stress

$s_{\max}$ maximum stress

$S_0$ fatigue limit
\(\bar{S}_o\) mean value of fatigue limit

\(S_{0p}\) safe fatigue limit

\(S_f\) scatter factor

\(t\) statistical variable

\(t_{\gamma}\) \(\gamma\) percentile of t-distribution

\(T\) fatigue life

\(T_p\) safe fatigue life, or probable number of cycles at which visually detectable cracks are formed

\(T_p^*\) probable fatigue crack propagation, or probable number of cycles at which the critical crack length is reached, i.e. failure is imminent

\(T_{50}\) fatigue life with a 50 percent reliability level

\(\hat{T}_{50}\) median value of test life

\(u_p\) standard normal deviator corresponding to the reliability level of \(p\)

\(u_{\gamma}\) standard normal deviator corresponding to the confidence level of \(\gamma\)

\(W\) width of plate

\(\bar{x}\) mean value of sample

\(\alpha\) shape parameter of \(p - s_a - s_m - N\) surface

\(\alpha_o\) plane stress/strain constraint factor

\(\alpha(a)\) geometry function of fatigue crack

\(\beta\) correction coefficient of the standard deviation

\(\gamma\) confidence level

\(\mu\) population mean value

\(\phi\) diameter of hole

\(\sigma\) static stress

\(\sigma_o\) population standard deviation
\( \sigma_u \) ultimate strength

\( \sigma_s \) standard deviation of logarithmic fatigue limit

\( \sigma_{0.2} \) yield stress

1 INTRODUCTION

It has been reported that 80-90\% of failures in steel structures are related to fatigue and fracture\(^1,2\). Therefore, fatigue reliability analyses now are widely used because of the requirement of safe operation of mechanical structures\(^3,4\). Fatigue loading on engineering structures results in onset of damage which, from time to time, will require repair. This can be expensive if the structure/artefact has to be taken out of service for the repair to be effected. Occasionally, if the damage is not identified at an early stage, there is a likelihood of sudden, catastrophic failure. Thus it is important for active reliability design to determine exactly the service life and inspection periods in order to ensure safety. From practice, it is proved that because of the random nature of external loading on structure\(^5\) and the internal heterogeneity of the structural material and manufacturing variabilities\(^6\), for the same style of structure under the same condition, the full-lives display large variations. Obviously, it is difficult for a deterministic methodology to evaluate the service life of the product sample and to include the randomness above mentioned. Thus there is a need for probabilistic approaches through a combination of probabilistic statistics and mechanics.

In order to guard against failures from unforeseen circumstances, two major approaches to structural substantiation have been devised. One is fatigue analysis and testing programme that attempts to establish a ‘safe life’ for the structure under assumed loading conditions\(^7\). This procedure implies that life can be predicted and that before the end of this time, the structure can be inspected, repaired and restored or retired from service. If the analysis can be established fairly early on in the design process, then any deficiencies can be eliminated or minimised. It has also been recognised that inevitably some structural damage and failures would occur and that catastrophic failure is almost never tolerable. This has led to approaches that are termed ‘damage tolerant’ or ‘fail safe’ designs\(^8\), in which the damage in designs/artefacts would be temporarily tolerated until repair can be effected or the damage assumes potentially critical dimensions. Both approaches are of interest and have been complementary to each other. However, at present, only a few studies have been reported regarding
the durability mathematical model incorporating both approaches, and it is necessary for the integrated and practical active reliability design system to be developed to assess structural economic first inspection period.

The overall aim of the work is to establish an integrated and practical durability model incorporating the safe life and damage tolerance design approaches for active reliability design system. Specific objectives are:

a) To present practical scatter factor formulae to determine the safe fatigue crack initiation and propagation lives from the result of a single full-scale test of a complete structure

b) To develop theoretical prediction techniques to establish the relationship equation between safe fatigue crack initiation and propagation lives with a specific reliability level using the two-stage fatigue damage cumulative rule

c) To derive a durability mathematical model incorporating the safe life and damage tolerance design approaches to assess the most economical first inspection period

d) To apply all above proposed methods/models to assess the structural economical life and to verify the feasibility and validity.

2 SCATTER FACTOR METHOD

The structural safe life of an engineering artefact with a high probability of survival and with a high confidence level is determined from the complete set of fatigue test data by means of scatter factor method[^9]. The scatter factor have been presented in probabilistic terms as the ratio between the best estimator of the fatigue life obtained in a small number of tests and the time to the first failure in a fleet of aircraft of specified size, at a specified reliability level and the scatter factor $S_f$ defined by sample median life is

$$ S_f = \frac{\hat{T}_{50}}{T_p} = 10^{-\mu/\sigma} \quad (1) $$

Equation (1) assumes a log-normal distribution for fatigue life $T$. Note that the sample median life $\hat{T}_{50}$ is different from the population median life $T_{50}$. Thus, a confidence level is introduced into the scatter factor so as to prevent the predicted value of the service life for the sample from inclining to the high side. The scatter factor is written as follows[^9]:

[^9]:
\[ S_f = 10^{\frac{u_u - u_r}{\sqrt{\frac{n}{n-1}}} \sigma_0} \]  

(2)

where \( \sigma_0 \) is the population standard deviation of logarithmic fatigue life. For the different types of metal material, the population standard deviation \( \sigma_0 \) of logarithmic fatigue crack initiation life has been recommended to be \( \sigma_0 = 0.16 \sim 0.20 \), while the population standard deviation \( \sigma_0^* \) of logarithmic fatigue crack propagation life has been suggested to be \( \sigma_0^* = 0.07 \sim 0.10 \). Equation (2) has been widely used, but its application conditions are: (a) the fatigue test results of full-scale structure must be complete data; and (b) the population standard deviation is given, \( \sigma = \sigma_0 \).

It is often the case that full-scale fatigue test results are incomplete because of cost and time constraints. Thus it is desirable to have a technique that accounts for the incompleteness of the dataset. Consider, as an example, the case of the nominally identical port and starboard wings of an aeroplane undergoing fatigue testing. If either of the two parts fails under a given load spectrum, the testing stops. The service life of the structure can be determined from this incomplete dataset using the following scatter factor.

\[ S_f = 10^{\left( \frac{u_u - u_r - 0.4308}{\sqrt{n}} \right) \sigma_0} \]  

(3)

When the population standard deviation \( \sigma_0 \) is unknown, it is possible to determine the service life approximately using the following scatter factor

\[ S_f = 10^{u_u \beta - t - L \left( \frac{1}{n+u_r} - \beta^2 - 1 \right)} \]  

(4)

3 TWO-STAGE MINER RULES TO PREDICT PROBABILITY FATIGUE CRACK INITIATION AND PROPAGATION LIVES

The structural safe life of an engineering artefact with a high probability of survival can be also predicted from the experimental data of fatigue behaviour and the fatigue loading history using two-stage fatigue damage cumulative theory (or Miner rule).

In the context of probabilistic fatigue life, \( N_{pi} \), this is written as:
\[ T_p = 1 / \sum_{i=1}^{k} \frac{n_i}{N_{pi}} \]  \hspace{1cm} (5)

where \( N_{pi} \) is the cycle number to fatigue crack initiation under the independent loading of \( i \)th cyclic stress pertinent to a reliability level of \( p \). Under the cyclic loading of sequential and stochastic spectra, Equation (5) can be written into an integral form:

\[ T_p = \frac{1}{\int_{0}^{(s)_{\text{max}}} \frac{n_i f(s)}{N_p(s)} ds} \]  \hspace{1cm} (6)

It is well-known that the cyclic stress level in a block of load spectrum is dominated by the two-parameter nominal stress \( (s_a, s_m) \), Equation (6) can be therefore extended to the two-parameter stress forms as

\[ T_p = \frac{1}{\sum_{i=1}^{k} \frac{n(s_a, s_m)}{N_p(s_a, s_m)}} \]  \hspace{1cm} (7)

\[ T_p = \frac{1}{\int_{(s_a)_{\text{max}}}^{(s_a)_{\text{max}}} \int_{(s_m)_{\text{max}}}^{(s_m)_{\text{max}}} \frac{n_i f(s_a, s_m)}{N_p(s_a, s_m)} ds_a ds_m} \]  \hspace{1cm} (8)

It is worthy to notice that, (1) using rain-flow count\footnote{Rain-flow count is a method for analyzing stress cycles in an actual load history.}, all stress cycles can be extracted from an actual load history and the statistical properties of stress cycle can be analyzed to determine \( n(s_a, s_m) \), \( f(s_a, s_m) \) and \( n_i \); (2) \( N_p(s_a, s_m) \) can be determined from the fatigue crack initiation \( p - s_a - s_m - N \) surface as

\[ N_p = C_p \left( \frac{\sigma_b - s_m}{\sigma_b - s_a} s_a - S_{0p} \right)^{m_p} \]  \hspace{1cm} (9)

where \( C_p \), \( m_p \) and \( S_{0p} \) are the undetermined parameters. Equation (9) can be transformed into the following form.

\[ \left( \frac{\sigma_b - s_m}{\sigma_b - s_a} s_a \right) = \frac{A}{N_p^\alpha} + 1 \]  \hspace{1cm} (10)

with

\[ \alpha = -1/m_p \]

\[ A = C_p^{\alpha} / S_{0p} \]
where $\alpha$ and $A$ is defined as the shape parameters of the $p - s_a - s_m - N$ surface. It has been reported that, for the same material or structure, the shape parameters $\alpha$ and $A$ of $p - s_a - s_m - N$ surface pertinent to the different reliability levels are approximately equal. Moreover, $\alpha$ and $A$ of the alloyed steel are 0.80632 and 4453 respectively, while those of the aluminium alloy equal 0.30135 and 64.

The safe fatigue limit $S_{0p}$ can be usually determined by means of the reduction factor formulation for fatigue strength as

$$r_s = \frac{S_{0p}}{S_0} = 10^{\left(\frac{u_p - u_d}{\sqrt{\sigma_s}}\right)\sigma_s}$$ \hspace{1cm} (11)

In case of that the fatigue strength follows a log normal distribution, the mean $\overline{S}_0$ and standard deviation $\sigma_s$ are calculated according to the following equations:

$$\log \overline{S}_0 = \frac{\sum_{i=1}^{n} \log S_{0i}}{n} \hspace{1cm} (12)$$

$$\sigma_s = \sqrt{\frac{\sum_{i=1}^{n} \left(\log S_{0i} - \log \overline{S}_0\right)^2}{n-1}} \hspace{1cm} (13)$$

The standard deviation of logarithmic fatigue limit of the alloyed steel $\sigma_s = 0.045$, while that of the aluminium alloy $\sigma_s = 0.06$. Substituting Equation (11) into Equation (10), one has

$$\frac{\sigma_b}{\sigma_b - s_m} \left\{ \frac{u_p - u_d}{\sqrt{\sigma_s}}\right\} = \frac{A}{N_{pa}} + 1$$ \hspace{1cm} (14)

It is also well-known that the fatigue process includes the two-stages of crack formation and crack propagation. One has extended the Miner rule to predict the probable number of cycles at which the critical crack length is reached, i.e. failure is imminent. Let $T_{p\ast}$ denotes the probable fatigue crack propagation, $T_{p\ast}$ can be calculated by means of the following equations.

$$T_{p\ast} = \frac{1}{k} \sum_{i=1}^{k} \frac{n(s_a, s_m)}{N_{pa}(s_a, s_m)}$$ \hspace{1cm} (15)
\[
T_p^* = \frac{1}{N_p} \int_{(s_{a})_{\min}}^{(s_{a})_{\max}} \int_{(s_{m})_{\min}}^{(s_{m})_{\max}} \frac{n_f f(s_a, s_m)}{N_p^*(s_a, s_m)} ds_a ds_m
\]

(16)

where \( N_p^*(s_a, s_m) \) can be determined from the fatigue crack propagation \( p - S_a - S_m - N \) surfaces as

\[
N_p^* = \frac{1}{f(s_a, s_m, a, p)} \int_{s_{a}}^{s_{a_{\max}}} da
\]

(17)

with

\[
\frac{da}{dN_p} = f(s_a, s_m, a, p)
\]

(18)

Equation (18) represents the crack propagation law in conformity with traditional approaches. Because the comprehensive influences of stress ratio \( r \), fracture toughness \( K_{IC} \) and fracture threshold value \( \Delta K_{th} \) are taken into consideration, the generalized Forman formula \(^{[11]}\) with 4-Parameter has been widely applied to describe the full range crack growth law.

\[
\frac{da}{dN} = C \left( \frac{1-f}{1-r} \right)^{m_1} \left( \frac{1-\Delta K}{\Delta K} \right)^{m_2} \left[ 1 - \frac{\Delta K}{(1-r)K_{IC}} \right)^{m_3}
\]

(19)

where \( C \), \( m_1 \), \( m_2 \) and \( m_3 \) are the undetermined parameters, \( f \) is fatigue crack opening function, which can be determined as

\[
f = \frac{K_{open}}{K_{max}} = \begin{cases} 
\max \left( r, A_0 + A_1 r + A_2 r^2 + A_3 r^3 \right) & r \geq 0 \\
A_0 + A_1 r & -2 \leq r < 0
\end{cases}
\]

(20)

with

\[
A_0 = (0.825 - 0.3\alpha_0 + 0.05\alpha_0^2) \left[ \cos \left( \frac{\pi s_{max}}{2 \sigma_0} \right) \right]^{\frac{1}{\alpha}}
\]

\[
A_1 = (0.415 - 0.071\alpha_0) \frac{s_{max}}{\sigma_0}
\]

\[
A_2 = 1 - A_0 - A_1 - A_3
\]

\[
A_3 = 2A_0 + A_1 - 1
\]
where $\alpha_0$ is the plane stress/strain constraint factor, and $s_{max}/\sigma_0$ is the ratio of maximum stress to the flow stress. By means of the randomization method of the deterministic equation, from Equation (21), it is possible to have the full range crack growth rate $p - da/dN - \Delta K$ curve.

\[
\frac{da}{dN_p} = C \times 10^{-\sigma_0 u_p} \cdot \left[ \left( 1 - \frac{f}{1 - r} \right) \Delta K \right]^{m_1} \frac{1 - \frac{\Delta K_{th}}{\Delta K}}{1 - \frac{\Delta K}{(1 - r)K_{1c}}}^{m_2}\]  

(21)

where $\sigma_0$ is the population standard deviation of logarithmic crack growth rate.

According to elastic fracture mechanics, in general, the stress intensity factor range $\Delta K$ is

\[
\Delta K = 2Y(a)s_a
\]

with

\[
Y(a) = a(a)\sqrt{\pi a}
\]

(22)

(23)

Substituting Equation (22) into Equation (21), the full range crack growth rate $p - da/dN - \Delta K$ curve can be expressed by using $s_a$ and $s_m$.

\[
\frac{da}{dN_p} = \frac{C \left\{ (1 - f)(s_a + s_m) \right\}^{m_1} \left[ Y(a) \right]^{m_2}}{\left[ 1 - \frac{s_a + s_m}{K_{1c}} Y(a) \right]^{m_2}}
\]

(24)

Separating the variables of Eqn. (24) and integrating, the fatigue crack propagation $p - S_a - S_m - N$ surfaces can be expressed as

\[
N_p = \frac{10^{\sigma_0 u_p} \cdot (2s_a)^{m_2}}{C \cdot \left\{ (1 - f)(s_a + s_m) \right\}^{m_1}} \int_{a_o}^{a} \left[ Y(a) \right]^{m_2} \cdot \left[ 2s_a Y(a) - \Delta K_{th} \right]^{m_2} \frac{da}{1 - \frac{s_a + s_m}{K_{1c}} Y(a)}
\]

(25)

From Equation (24) and fracture toughness $K_{1c}$, it is possible to determined the critical crack length $a_{cr}$, or

\[
K_{1c} = \alpha(a_{cr})\sigma\sqrt{\pi a_{cr}}
\]

(26)
4 DURABILITY MATHEMATICAL MODEL INCORPORATING SAFE LIFE METHODOLOGY AND DAMAGE TOLERANCE APPROACH

Because the fatigue process includes the two-stages of crack formation and crack propagation, it is necessary for structural life to be assessed using a combination of safe life methodology and damage tolerance approach. The safe life method is applied for life assessment and damage tolerance approach is for operational assessment of safety. So, a life assessment model can be developed for a finite life design. An approach could be based using probable lives at damage inception and when cracks reach critical dimensions. So the most economical first inspection period $T_i$ should satisfy the condition of $T_p = T_p^*$; the condition $T_p < T_p^*$ implies that the structure does not need to be inspected as yet.

As is above mentioned, the probable fatigue crack initiation and propagation lives $T_p$ and $T_p^*$ of a structure can be determined from a small number of experimental results of using scatter factor formulations (2) to (4), also be calculated by means of the equations (7), (8), (15) and (16). $T_p$ and $T_p^*$ can be written as

$$p = f(T_p) \quad (27a)$$
$$p^* = g(T_p^*) \quad (27b)$$

The probabilities of fatigue crack initiation and propagation are $(1 - p)$ and $(1 - p^*)$ respectively. Only when a fatigue crack appears and grows to a critical value does structural failure occur; so the structural risk of failure and reliability are $(1 - p)(1 - p^*)$ and $1 - (1 - p)(1 - p^*)$ respectively. For a specific structural reliability of $R$ to ensure the safety of structure, one has

$$R = 1 - (1 - p)(1 - p^*) \quad (28)$$

Substituting Equations (27a) and (27b) into (28), one obtains

$$R = f(T_p) + g(T_p^*) - f(T_p) \cdot g(T_p^*) \quad (29)$$

From Equation (29) and the condition of $T_p = T_p^*$ satisfied by the most economical first inspection period, $T_i$ can be determined by the following equation.
\[ R = f(T_i) + g(T_i) - f(T_i) \cdot g(T_i) \]  

Equation (30) represents the outcome of the durability model to determine the most economical first inspection period. A model such as the one outlined above for a structural component could be extended to cover a large or whole structure comprising several components. Such an extension will require the formulation of appropriate linkage or design rules between safe lives of individual components and their influence on adjacent components and then on the whole structure. Component and large structure testing will need to be carried out to validate the developed approach. For a large or whole structure comprising several components, if failure of any component will result in the failure of whole structure, then the reliability of whole structure becomes

\[ R = \prod_{i=1}^{m} \left[ 1 - (1 - p_i)(1 - p_i^*) \right] \]  

Substituting Equations (27) into (31), one has an expression to determine the most economical inspection period

\[ R = \prod_{i=1}^{m} \left[ f_i(T_{pl}) + g_i(T_{pl}^*) - f_i(T_{pl}) \cdot g_i(T_{pl}^*) \right] \]  

5 APPLICATION OF CONCEPTS

A fastening structure at the root of helicopter blade consists of the auricle junction, bolts and the blade beam: the auricle junction and bolts are made of 40CrNiMoA alloyed steel while the blade beam is made of LD2 aluminium alloy. The geometry and dimensions of fastening structure are illustrated in Fig. 1. From the engineering practice, it is clear that the first fatigue failure of fastening structure appears on the auricle junction, it is therefore important to assess the economic first inspection period of the auricle junction. The mechanical behaviour of 40CrNiMoA alloyed steel are as follows: Young’s modulus \( E = 204 \) GPa, Poisson’s ratio \( \nu = 0.3 \), ultimate strength \( \sigma_u = 1080 \) MPa, yield stress \( \sigma_{0.2} = 880 \) MPa, fracture toughness \( K_{IC} = 4691 \) MPa\(\sqrt{\text{mm}} \), fracture threshold value \( \Delta K_{th} = 342 \) MPa\(\sqrt{\text{mm}} \), mean fatigue limit \( S_0 = 75.4 \) MPa and standard deviation of logarithmic fatigue limit \( \sigma_s = 0.045 \). The population standard deviation of logarithmic crack growth rate \( \sigma_0 = 0.2 \). From Equations (14) and (21), the \( p - s_m - s_a - N \) surface and \( p - da/dN - \Delta K \) curve of
the auricle junction are respectively given as

\[ N_p = \frac{33497}{\left( \frac{1080.0 - s_m}{75.4 \times 10^{0.645 s_u}} \right) - 1}^{1.2402} \]  

\[ \frac{da}{dN} = 3.62 \times 10^{-0.25 a} \times \left( 1 - f \right)^{1.715} \left( 1 - \frac{342.0}{\Delta K} \right)^{2.314} \left( 1 - \frac{\Delta K}{4691.0 \times (1 - r)} \right)^{0.285} \]  

Using the finite element method (FEM), the nominal stress spectrum (shown in Fig. 2) can be obtained from the actual load spectrum. A block of the nominal stress spectrum represents the actual load history of one flight hours. Consequently, by means of Equation (9), the safe fatigue crack initiation lives with a series of reliability level are calculated from the nominal stress spectrum (shown in Fig. 2) and the \( p - s_m - s_u - N \) surface formulation (33) of the auricle junction. The calculations are shown in Table 1.

Fatigue crack growth mode of auricle junction can be simulated into a single side-penetrated crack along the notched edge of a finite-width plate subjected a cyclic from the bolt (shown in Fig. 3), the geometry function of fatigue crack of auricle junction is then determined as

\[ \alpha(a) = \left( 0.707 - 0.18 \lambda + 6.55 \lambda^2 - 10.54 \lambda^3 + 6.85 \lambda^4 \right) \cdot \frac{\pi \phi}{2W} + \frac{1}{\pi} \left( \frac{\phi}{\phi + a} \right) \sqrt{\lambda} \]

\[ \cdot \sqrt{\sec \left( \frac{\pi \cdot \phi + a}{2 \cdot W - a} \right) \cdot \sec \left( \frac{\pi \phi}{2W} \right)} \]

with

\[ \lambda = \frac{\phi}{\phi + 2a} \]

The critical crack length \( a_c \) of auricle junction is calculated to be 12.0 mm from fracture toughness \( K_{ic} \) using Equation (30). According to the visually detectability of crack in engineering practice, the initial crack size is chosen to be 1.25 mm. Substituting Equations (34) into (25), the fatigue crack propagation \( p - S_a - S_m - N \) surfaces of auricle junction can be expressed as
\[ N_p = \frac{10^{(0.2 \nu_r)} \cdot (2s_a)^{2.314}} {3.62 \cdot (1 - f)(s_a + s_m)]^{0.715} \int_{12.0}^{25} \frac{[2s_a Y(a) - \Delta K_{ic}^{2.314}]^{0.599} \left[ \frac{1 - s_a + s_m}{K_{ic}} Y(a) \right]}{0.285} da \]  

(36)

As a result, using Equation (15), the safe fatigue crack propagation lives with a series of reliability level are determined from the nominal stress spectrum (shown in Fig. 2) and the NSSp curve equation (36) of the auricle junction. The calculations are shown in Table 2.

For a high specific reliability level of 0.999999 to ensure the safety of auricle junction, or \( R = 0.999999 \), from Equation (28), one has

\[ 0.999999 = 1 - (1 - p)(1 - p^*) \]  

(37)

Using Equation (37), the relationship curve between the safe fatigue crack initiation and propagation lives can be obtained from Tables (1) and (2) and shown in Fig. 4. From Fig. 4, it is easy to obtain the most economical first inspection period \( T_i \), namely, when \( T_i = 200 \) flight hours, the condition of \( T_p = T_p^* \) can be satisfied. As demonstrated in above application example, using the durability mathematical model incorporating safe life methodology and damage tolerance approach, the most economical first inspection period can be obtained realistically and easily.

6 CLOSURE

The focus of this paper has been to develop a new and practical durability mathematical model incorporating safe life methodology and damage tolerance approach to assess structural economical first inspection period. The applicability of the new model has been shown for a fastening structure at the root of helicopter blade for estimating the most economical first inspection period.

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REFERENCES


Fig. 1  Fastening structure at the root of helicopter blade (all dimension in mm)

Fig. 2  Nominal stress spectrum
Fig. 3 Crack growth model for auricle junction

Fig. 4 Relationship curve between safe fatigue crack initiation and propagation lives

Table 1 The calculations of safe fatigue crack initiation life

<table>
<thead>
<tr>
<th>Reliability level $p$</th>
<th>Safe crack initiation life $T_p$ (flight hour)</th>
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<td>Reliability level $p^*$</td>
<td>Safe crack propagation life $T_{p^*}$ (flight hour)</td>
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</tbody>
</table>

Table 2 The calculations of safe fatigue crack propagation life