PROBABILISTIC RELIABILITY ASSESSMENT OF A STEEL FRAME APPLYING THE SBRA METHOD

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ABSTRACT

This paper deals with the reliability assessment of steel planar frame structures using the probabilistic SBRA method. The simulation-based approach allows for the consideration of load effects combinations, the effects of global and local imperfections and the variations of mechanical and geometrical properties of individual structural components. In this connection attention is paid to an alternative way of the local and global stability check based on the SBRA method applying the second order theory. Using a pilot example the proposed probabilistic approach to the assessment of the safety and serviceability of frames is illustratively explained.

1. INTRODUCTION

This paper is concerned with the reliability assessment of steel structures using the probabilistic SBRA method documented in the book [1]. Advances in computer technology make it possible to utilize the potential of the SBRA method not only for the reliability assessment of simple structures and their components [2], but also for the reliability assessment of more complex, statically indeterminate frame structures, see e.g. [3]. In connection with the transition to a probabilistic reliability assessment of systems, it is necessary to select appropriate reference values, which serve to define the resistance limits applied in the calculation of the probability of failure P_f of the structure and its components. Next it is necessary to evaluate the suitability of particular transformation models serving as tools to transform the loading into the response of the structure to the load with regard to the substance of the stochastic method. The relation and interaction of imperfections should be also considered and an appropriate transformation model selected. In this connection attention is paid to the transition from the traditional assessment of buckling strength in compliance with contemporary standards (based on determining buckling lengths and buckling factors) to the SBRA "strength stability concept" using 2nd order theory.
The problems of structural stability of frames are illustratively explained by safety and serviceability assessment of a steel planar frame which is exposed to a system of vertical and horizontal mutually uncorrelated actions, see Figure 1. A brief study concerning the influence of the frame bearing (ideally rigid or pinned) on resulting reliability is given at the end of this paper.

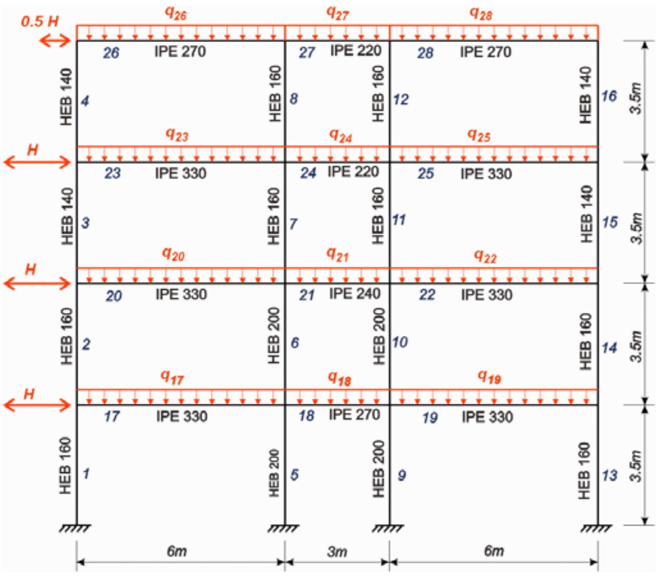

Figure No.1: The scheme of the frame

2. EXAMPLE OF RELIABILITY ASSESSMENT OF THE FRAME

The problems of safety and serviceability assessment of steel frame structures are described and discussed in following chapter using an example of the probabilistic reliability assessment of a four-story three bay moment resisting steel frame, see Figure 1.

2.1 ASSIGNMENT AND UNDERLYING ASSUMPTIONS

The scheme of the planar steel frame exposed to the system of horizontal forces and vertical uniformly distributed loadings is shown in

Figure 1. It is assumed that the material properties (yield stress), the geometrical properties (initial imperfections) and the actions are mutually non-correlated random variables. The length of individual bars and their cross-sectional characteristics are considered to be constant. It is believed that the structure is secured against buckling out of its plane. The column bases are considered as ideally rigid, beam-to-column joints are also contemplated as ideally rigid.

The actions on the structure are considered mutually independent and are represented, according to the SBRA method, by the load duration curves and corresponding histograms (see Table 1). The histograms used in Table 1 are taken from the database contained in the book [2]. Considering the uniformly distributed actions, the same value of the dead load is simulated for the whole story. It means for example, that the simulated value of the action q_{17}*_dead* equals the value of the action q_{18}*_dead* and q_{19}*_dead* (dead load of the beams above the first story). The values of other uniformly distributed actions (short lasting load, long lasting load, snow load) are all statistically independent (the simulated value of the action q_{17}*_short* does not equal the value of the action q_{18}*_short* and q_{19}*_short*).

Table No.1: Loading data according to SBRA

Loading		***Maximum value***	***Used Histogram***
Symbol	***Description***		
q_{17} , q_{19} q_{20} , q_{22} q_{23} , q_{25}	Dead load Short lasting load Long lasting load	20 kN/m 15 kN/m 20 kN/m	Dead1.dis Short1.dis Long1.dis
q_{18} , q_{21}, q_{24}	Dead load Short lasting load Long lasting load	20 kN/m 30 kN/m 10 kN/m	Dead1.dis Short1.dis Long1.dis
q_{26} , q_{27}, q_{28}	Dead load Snow load	20 kN/m 20 kN/m	Dead1.dis Snow2.dis
H	Wind load	45 kN	Wind1.dis

The columns of the frame are hot-rolled sections HEB, the beams of the frame are hot-rolled sections IPE, see Figure 1. The steel grade S235 is used. The yield stress F_y of the steel is represented by a histogram *DS235Fy01.dis* proposed by Rozlívka & Fajkus, for more information see [4].

The allowance for imperfections in the analysis of frames is intended to cover effects such as lack of verticality, lack of straightness, perpendicularity (the quality of being at right angles to a given line or plane), effects of residual stresses, etc. These effects could be expressed in compliance with Eurocode 3 [5] by means of a set of equivalent global and local geometrical imperfections. These imperfections are not actual construction tolerances but, because they are intended to represent the effect of a number of factors, are likely to be larger than such tolerances.

The equivalent geometrical imperfection in the form of the initial global sway imperfection Φ is introduced using a bounded histogram corresponding to a normal distribution $N(\mu = 0, \sigma = 1 / 1141)$ and considering the range limitations $\Phi_{Max} = \pm 1 / 380$ (rad). The value Φ_{Max} is estimated in line with Eurocode 3 [5] for the frame shown in Figure 1.

The bar imperfections may be substituted by one equivalent local geometric imperfection in the form of initial local bow imperfection with maximal mid-span deflection e_o. Using the SBRA method, the initial local bow imperfection of the frame columns is represented by a bounded histogram of normal distribution $N(\mu = 0, \sigma = 0.0047)$ with the range limitations $e_{0.Max} = \pm 0.014$ (m). The initial local bow imperfection of the outer bay beams is represented by a bounded histogram of normal distribution $N(\mu = 0, \sigma = 0.0067)$ with the range limitations $e_{0.Max} = \pm 0.02$ (m). The initial local bow imperfection of the inner bay beams is represented by a bounded histogram of normal distribution $N(\mu = 0, \sigma = 0.0033)$ with the range limitations $e_{0.Max} = \pm 0.01$ (m). The values $e_{0.Max}$ are considered in line with the recommendations given in Eurocode 3 [5].

2.2 DESIGN PROCEDURE

The following chapter shortly describes the main principles and procedures of the probabilistic SBRA method with respect to the reliability assessment of the frame structure. The attention is paid to the interaction of the randomly variable quantities (it is solved interaction of 73 randomly variables), next to the introduction of imperfections to the structure analysis and to the probabilistic evaluation of structure safety and serviceability.

Using the SBRA method, in each simulation step a new combination of random variables is generated (including variable actions, imperfections, yield stress). This procedure makes it possible to perform the global analysis according to the second order theory including the effects of all equivalent geometrical imperfections on the response of the structure. If second order effects in individual members and relevant member imperfections are totally accounted for in the global analysis of the structure, no individual stability check for the members is necessary. The reliability check of the frame structure is in this way significantly simplified. Such an approach is, unfortunately, inapplicable by the analysis of more complex frame structures using the deterministic methods (Partial Factors Method [6]) due to an excessive increase of imperfection combinations. Taking into account all global and local imperfections, the number of imperfection combinations is described by an equation $2^{(n+1)}$, where n is the number of frame bars. The number of imperfection combinations for the checked frame shown in Figure 1 is approximately $5x10^8$ combinations! In addition, these imperfection "states" should be combined next with combination of loadings specified in section 2.1. The combinations formulas given in Eurocodes [6] serve only to combine the actions but these formulas do not describe how to combine the imperfections. It is evident that such approach according the partial factors method is not optimal. On the other hand, the aforementioned procedure may be efficient using simulation probabilistic methods.

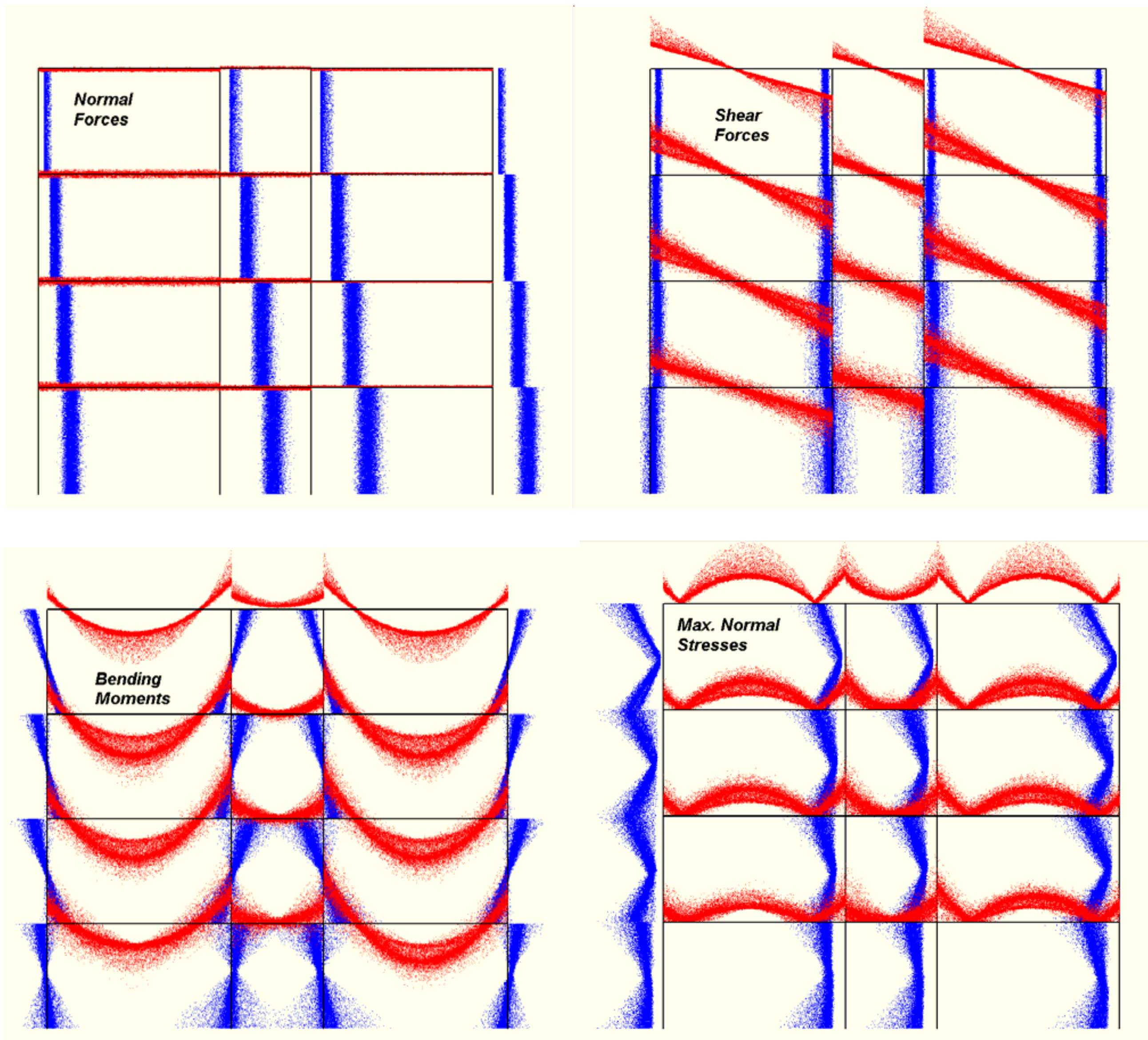

Figure No.2: The scatter of normal forces, shear forces, bending moments and maximum values of normal stresses

The safety assessment using the SBRA method is expressed by comparing the calculated probability of failure P_f (using the Monte Carlo simulation technique) with the target probability P_d for safety assessment given, for instance, in the Czech code [7]. Considering the reference value defined in this example by the onset of yielding, i.e. the probability of failure of individual bars equals the probability of exceeding the yield stress F_y in the most loaded fibres of the investigated cross section of the bar, the safety condition is expressed by

$$P_f = P\left[(F_y - \sigma) < 0\right] < P_d \,. \qquad (1)$$

Assuming an elastic response of the structure to the loading, the maximum stress σ in the outer fibres of the section can be determined by the equation

$$\sigma = abs\left(\frac{M}{W}\right) + abs\left(\frac{N}{A}\right), \qquad (2)$$

where M (kNm) and N (kN) are the values of internal forces at the checked section, while W (m3) and A (m2) are the cross-sectional geometrical properties. Values M and N are calculated using transformation model based on the second order theory, whereas the influences of global and local imperfections are accounted for in the analysis.

To enable a visual control of performed structural analysis, the scatter of resulting internal forces, bending moments and maximum normal stresses can be illustrated using the set of dots, so called “anthills” and “ants”, corresponding to the random combinations of input variables, see Figure 2. The results shown in Figure 5 were obtained using the MCD 1.0 program [8].

Analyzing the equation (2) with help of the Monte Carlo simulation, the probability of failure in any particular cross section can be ascertained. The calculation of the probability of failure P_f, referring to individual cross-sections, leads to a plot of so-called “probability of failure curves” (see Figure 3). In this figure, the P_f curves refer to the second order theory. The P_f curves indicate that the sections controlling the safety are at the ends of the columns and beams. Calculation was performed using the MCD 1.0 computer program during which 10 million simulation steps were made.

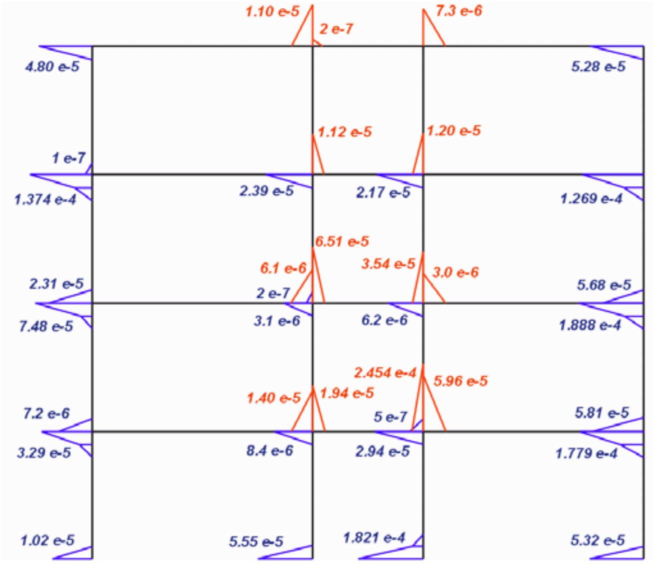

Figure No.3: The probability of failure curves - safety

The serviceability criterion related to the exceedance of the tolerable horizontal displacement $w_{\lim}$ of the top frame joints is expressed by an equation

$$P_f = P\left[(w_{\lim} - w) < 0\right] < P_d \,, \qquad (3)$$

where w is the value of resulting horizontal displacement of the top frame joints determined according to the second order theory, P_f is the calculated probability of failure and P_d is the target probability for serviceability assessment defined, for instance, by building owner or by the code (it differs from the target probability for safety assessment). In the solved example $w_{\lim}$ is represented by the value $w_{\lim} = H/200$, where H is the height of the frame. Using MCD 1.0 program, the calculated probability of failure equals $1.015*10^{-2}$.

The serviceability assessment of horizontal beams is related to the exceedance of the tolerable vertical deflection $w_{\lim.Beam}$. In the solved example $w_{\lim.Beam}$ is represented by the value $w_{\lim.Beam} = L/250$, where L is the length of the beam. The serviceability condition can be expressed by

$$P_f = P\left[(w_{\lim.Beam} - w_{Beam}) < 0\right] < P_d \,, \qquad (4)$$

where w_{Beam} is the maximum value of vertical beam deflection determined according to the second order theory.. The calculated probabilities of failure referring to the vertical deflection of the beams can be also illustrated using probability of failure curves, see Figure 4. It can be seen that the sections with maximal vertical deflection and therefore with maximal value of probability of failure are located at the mid-span of the beams.

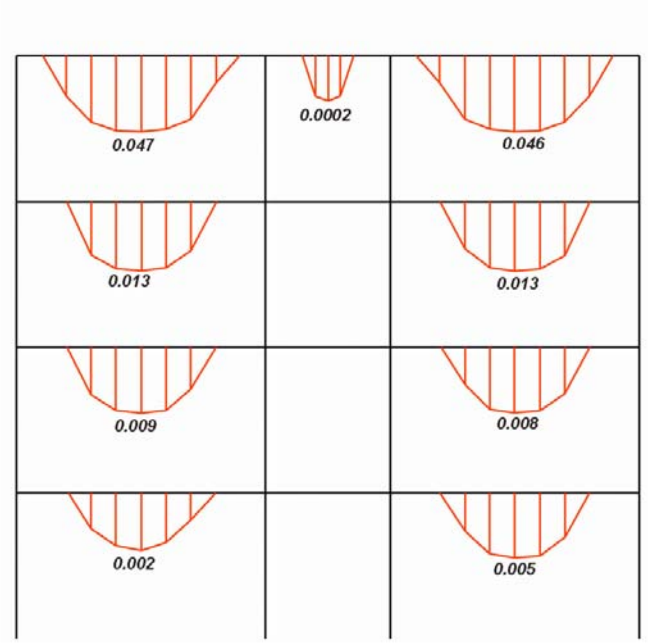

Figure No.4: The probability of failure curves -serviceability

3. A COMPARATIVE STUDY

The P_f curves may be very easy and effectively used to compare and evaluate different design alternatives. So as to demonstrate the potential of P_f curves, the reliability assessment of the frame described in the section 2.1 was performed, whereas two models of column bases were complemented:

1. ideally rigid bearing (see section 2)
2. ideally pinned bearing

The results of safety assessment are illustrated in Figure 5 using the corresponding "probability of failure curves". By comparing the results corresponding to both mentioned alternatives, important differences between calculated probabilities of failure can be observed predominantly at the bars lying within the first and second story. These bars are mostly influenced by the frame bearing (see the scatter of bending moments at the frame with pinned column bases, illustrated in Figure 6, and compare with results in section 2.2). In addition the effects corresponding to the second order theory are much greater at the frame with pinned column bases, see [9].

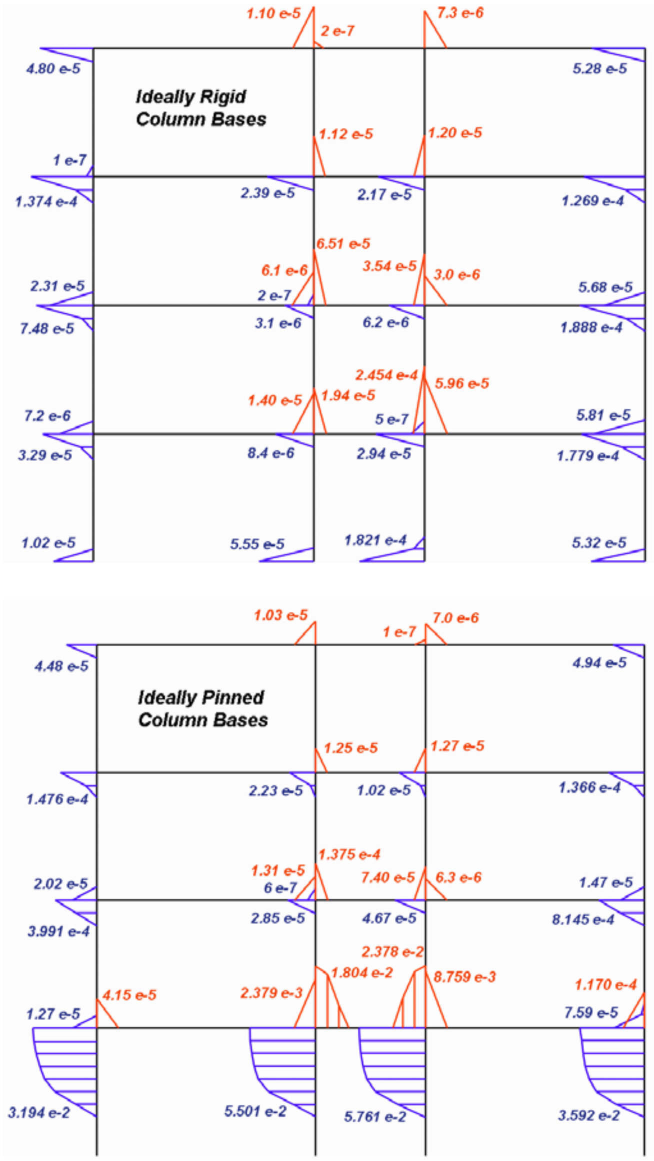

Figure No.5: The probability of failure curves - safety

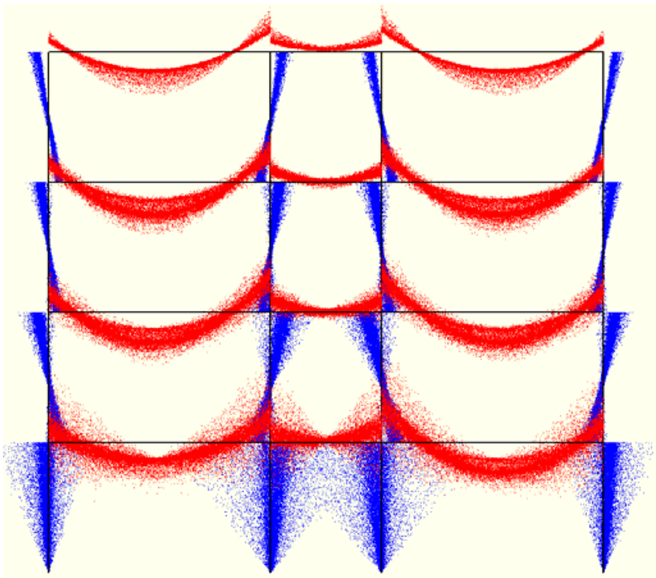

Figure No.6: The frame with pinned bearing – bending moments

With regard to the serviceability assessment, the probability of failure corresponding to the exceedance of the tolerable horizontal displacement $w_{\lim}$ of the top frame was calculated. The calculated probability of failure equals $P_f = 1.015*10^{-2}$ (in case of ideally rigid bearing) and $P_f = 9.127*10^{-2}$ (in case of ideally pinned bearing). This important difference is caused by the bearing influence and by different "sensitivity" of booth mentioned frames to the second order theory effects.

5. CONCLUSIONS

The transition from the pre-computer era to the era of advanced computer technology leads to introduction of qualitatively new probabilistic structural reliability assessment concepts. This paper turns attention to the application of one of the advanced concepts, the Simulation-Based Reliability Assessment method, (see SBRA [1]) applicable in codes and in designers' everyday work.

Using a pilot example the probabilistic safety and serviceability assessment of steel planar frame structures according to the SBRA method is explained and the assessment procedure is indicated. All input variables are expressed by non-parametric distributions [2]. The load effect combinations, the effects of global and local imperfections and the variation of mechanical and geometrical properties of individual structural components are evaluated using direct Monte Carlo technique. The second order theory transformation model applied in order to evaluate the response of the structure to the load combinations leads to non-traditional local and global stability check. The safety and serviceability criteria are checked by comparing the calculated probability of failure P_f and the target probability P_d given in code [7]. Such an approach allows for a complex assessment and optimization of the load carrying system.

Hundreds examples of the application of the SBRA method (safety, serviceability and durability of steel, concrete, wood and composite structures), databases of histograms, computer programs and more details are available in the textbooks [1] and [2], and on the web www.sbra-anthill.com.

ACKNOWLEDGEMENTS

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