A family of performance control policies of stochastic dynamic systems are proposed based on the probability density evolution theory and numerical optimizing strategies in the present paper. Firstly, the general form of the control policies of stochastic optimal control systems is raised according to the classical optimal control theory of linear quadratic regulator (LQR). Then, three classes of optimal control norms with objective performance indices are developed at hierarchical levels from the mean sub-norm, the mean-standard deviation sub-norm to the exceedance probability sub-norm. A linear single-degree-of-freedom system subjected to random ground motions is investigated for the illustrative purpose. The results show that the effectiveness of responses control hinges on the physical meanings of the optimal control norms. The multi-penalty norm, e.g., the acceleration- and input force constrained displacement controlled norm, in the objective performance norms behaves more comprehensively than the single-penalty norm and non-penalty norm that is the prime norm to the structural single-objective control. Among the three sub-norms the exceedance probability sub-norm is the most reasonable.

1. INTRODUCTION

In the classical stochastic optimal control theory, random excitations specifying disturbances and sensor noise are generally assumed to be white noise or filter white noise, and the corresponding stochastic optimal control methods, such as the linear quadratic Gaussian (LQG) control, minimum variance control and covariance control, are seeking for the optimal system performance in the ensemble-expected sense or in the mean square sense. To general stochastic dynamical systems, a family of probability density evolution method has been developed since 2002[1,2], and the generalized density evolution equation (GDEE) was formalized[3,4]. The GDEE profoundly reveals the essential ties between deterministic systems and stochastic systems by introducing physical relationships into stochastic systems. It can be naturally extended to the structural performance optimal controls of stochastic dynamic systems subjected to general random excitations.

Generally, there are two weighting matrices, Q and R, with three kinds of strategies being included. The first involves trial-and-error procedures, namely, one can assign the positive semi-definite weighting matrix Q, then check the stability condition and the performance objectives. The second strategy is a systematic way of assigning the weighting matrix by use of the Lyapunov stability condition[6]. The third strategy is the simply optimizing method, which searches for optimal weighting matrices based on the moments of extreme values of interested quantities related to the objective performance, e.g., the mean values of controlled quantities[7,8].

One could see that the above three classes of strategies of choosing weighting matrices are not optimal programmes, and they are, at most, approximately local optimal programmes. In this survey, a family of performance control policies of stochastic dynamical systems are proposed based on the probability density evolution theory and numerical optimizing strategies. Firstly, the general form of the control policies of stochastic optimal control systems is raised according to the classical optimal control theory of linear quadratic regulator (LQR). Then, three classes of optimal control norms with objective performance indices are developed at hierarchical levels from the mean sub-norm, the mean-standard deviation sub-norm to the exceedance probability sub-norm. A linear single-degree-of-freedom system subjected to
random ground motion is investigated for the illustrative purpose. The results show that the effectiveness of responses control hinges on the physical meanings of the optimal control norms. The multi-penalty norm, e.g., the acceleration- and input force constrained displacement controlled norm, in the objective performance norms behaves more comprehensively than the single-penalty norm and non-penalty norm that is the prime norm to the structural single-objective control. Among the three sub-norms the exceedance probability sub-norm is the most reasonable.

2. GENERAL FORM OF OPTIMAL CONTROL POLICIES

Consider a linear building structure of $n$-degree-of-freedom system equipped with active control systems and subjected to random excitations. The equation of motion is given by

$$\ddot{X}(t) + C\dot{X}(t) + KX(t) = BU(t) + DF(\Theta,t)$$  \hspace{1cm} (1)

where $X(t) = [X_1, X_2, \cdots, X_n]^T$ is an $n$th order vector, with $X_i(t)$ being the drift of the $i$th story; $U(t) = [U_1, U_2, \cdots, U_r]^T$ is an $r$th order vector, consisting of $r$ control forces, subscript $T$ denoting the transpose; $F(\Theta,t)$ is a physical stochastic excitation vector, $\Theta = [\Theta_1, \Theta_2, \cdots, \Theta_s]$ being the stochastic parameter vector characterizing the randomness from external excitation with the joint probability density function (PDF) of $p_\theta(\theta)$. $M$, $C$ and $K$ are $(n \times n)$ mass, damping and stiffness matrices, respectively; $B_s$ is an $(n \times r)$ matrix denoting the location of controllers; $D_s$ is an $(n \times p)$ matrix denoting the location of excitations.

In the state space, Eq. (1) can be rewritten as

$$\dot{Z}(t) = AZ(t) + BU(t) + DF(\Theta,t)$$  \hspace{1cm} (2)

with the initial condition $Z(t_0) = Z_0$, where $Z(t)$ is a $2n$th order state vector, $A$ is a $(2n \times 2n)$ system matrix; $B$ is a $(2n \times r)$ controller location matrix, and $D$ is a $2n$th order excitation location vector, respectively.

$$Z(t) = \begin{bmatrix} X(t) \\ \dot{X}(t) \end{bmatrix} ; \quad A = \begin{bmatrix} 0 & 1 \\ -M^{-1}K & -M^{-1}C \end{bmatrix} ; \quad B = \begin{bmatrix} 0 \\ M^{-1}B_s \end{bmatrix} ; \quad D = \begin{bmatrix} 0 \\ M^{-1}D_s \end{bmatrix}$$  \hspace{1cm} (3)

For a given $\Theta$, Eq. (2) is numerically tractable using time integration methods, and can give any required responses of the system being investigated, such as displacement, velocity, acceleration and control force.

The standard quadratic performance index of the optimal control depends on $\Theta$, which is given as Bolza form[9]

$$J(Z, U, \Theta, t ) = \frac{1}{2} Z(t)P(t)Z(t) + \frac{1}{2} \int_{t_0}^{t_f} [Z(t)QZ(t) + U(t)RU(t)]dt$$  \hspace{1cm} (4)

where $Q$ is a $(2n \times 2n)$ positive semi-definite matrix, $R$ is a $(r \times r)$ positive definite matrix, and $t_f$ is the duration time defined to be longer than that of the excitation. As should be noted, the performance index is defined as the ensemble-expected formula of Eq. (4) in the classical stochastic optimal control theory[10].

To minimize the performance index $J$ subjected to the constraint given by Eq. (2), the necessary conditions based on the Maximum Principles developed by Pontryagin in 1957, is applied. It exists for each sample $\theta$ in the sample space $\Theta$

$$U(t) = -R^{-1}B^T\lambda(t)$$  \hspace{1cm} (5)
where $\lambda(t)$ is a 2nd order vector representing the costate variables (or Lagrange multipliers), and could be adjusted according to state vector $Z(t)$ and excitation vector $F(\Theta,t)$ when a closed-open-loop control system with infinite-time optimal control is considered. Using Riccati optimal control method, the general form of control force vector of the closed-loop control system is

$$U(\Theta,t) = f(Q,R,\Theta)Z(t) \quad (6)$$

in which $f(\cdot)$ is the operator of optimal control policy.

One could see from Eq. (4) that the standard quadratic performance index $J(Z,U,\Theta,t)$ relies on the weighting matrices $Q$ and $R$. Generally, $Q$ and $R$ are determined by the comparative importance of the state vector $Z(t)$ and the control force vector $U(t)$ during minimizing the performance index. If the norm of $Q$ is larger than that of $R$, the emphasis is paid on $Z(t)$, otherwise on $U(t)$. Therefore, the choice and determination of $Q$ and $R$ is referring to as a sub-optimization, the goal of which is to get the optimal parameter vector $(Q^*,R^*)$ of optimal control policy $f(\cdot)$.

Further, it is seen that the state vector and the control vector both hinging on the stochastic vector $\Theta$ are governed by the following generalized density evolution equations (GDEE), respectively[3].

$$\frac{\partial p_{z\Theta}(z,\Theta,t)}{\partial t} + \dot{Z}(\Theta,t)\frac{\partial p_{z\Theta}(z,\Theta,t)}{\partial z} = 0 \quad (7a)$$

$$\frac{\partial p_{u\Theta}(u,\Theta,t)}{\partial t} + \dot{U}(\Theta,t)\frac{\partial p_{u\Theta}(u,\Theta,t)}{\partial u} = 0 \quad (7b)$$

The corresponding instantaneous probability density functions (PDFs) of $Z(t)$ and $U(t)$ could be obtained by

$$p_z(z,t) = \int_{\Omega_\Theta} p_{z\Theta}(z,\Theta,t)d\Theta \quad (8a)$$

$$p_u(u,t) = \int_{\Omega_\Theta} p_{u\Theta}(u,\Theta,t)d\Theta \quad (8b)$$

where $\Omega_\Theta$ is the distribution domain of $\Theta$; $\Theta$ is the sample of $\Theta$; the joint PDFs $p_{z\Theta}(z,\Theta,t)$ and $p_{u\Theta}(u,\Theta,t)$ are the solutions of Eqs.(7a), (7b) respectively.

The GDEEs, Eqs. (7a) and (7b) are one-order quasi-linear partial differential equations with initial and boundary condition, the numerical solving procedure of which involves: probability partition using the tangent spheres method[11], deterministic dynamic response analysis, difference schemes including the modified Lax-Wendroff scheme with TVD nature and its combination with the one-sided scheme[12,13]. It has been proved that the structural dynamic reliability assessment could be implemented by the GDEE when introducing the concept of virtual stochastic process in the extreme value distribution theory[13]. In addition, a Dirac-sequence approach, could be used in the dynamic reliability assessment so as to reduce the computational efforts[14].

3. OPTIMAL PERFORMANCE CONTROL NORMS OF STOCHASTIC SYSTEMS

The weighting matrices $Q$ and $R$ are both theoretically time-dependent but generally assumed to be time-independent. Some work is focused on the choices of the weighting matrices with symmetrical full-matrix forms. It is seen that the diagonal elements are generally far larger than the off-diagonal elements[5]. Therefore, it is reasonable to assume the weighting matrices to be diagonal, and the general forms of weighting matrices are[7]
The optimal weighting matrices choice in Eq.(9) is a multi-parameter optimization, which is difficult to be realized by the preceding three choice strategies of the weighting matrices, although they tend to be achieved by certain numerical optimizing strategy[15].

Since the state vector and the control force vector are governed by GDEE, respectively. Moreover, the GDEE reveals the physical evolution processes of stochastic systems with sample-path description. With this consideration, a family of optimal performance control polices could be developed according to the probability density evolution processes of interested quantities.

### 3.1 OPTIMAL PERFORMANCE CONTROL NORM BASED ON STATE VECTOR

Take the state vector as the control quantity, and construct an equivalent extreme-value process \( \max_{t} [Z_{i}(\Theta, t)] \), where \( t \) is the duration time of the stochastic process investigated; \( i \) being the unit. (The concept of the equivalent extreme-value process has been proved with rigorous mathematics and been proposed to evaluate the reliability of the system[16].

Define an equivalent extreme-value vector

\[
\tilde{Z}(\Theta) = \max_{t} \max_{i} [Z_{i}(\Theta, t)]
\]

Three sub-norms herein are proposed with hierarchical levels.

(a) Mean sub-norm

\[
\min (J') = \min \{ E[\tilde{Z}] \}
\]

The physical sense of the mean sub-norm is that the expectation of extreme value of the control quantity is minimized, as shown in Fig. 1.

(b) Mean-standard deviation sub-norm

\[
\min (J') = \min \{ E[\tilde{Z}] + r \times \sigma[\tilde{Z}] \}
\]

The physical sense of the mean-standard deviation sub-norm is that the expectation of the extreme value of the control quantity is minimized, and the deviation extent of which is within certain range, as shown in Fig. 2.

(c) Exceedance probability sub-norm

\[
\min (J') = \min \{ P(\tilde{Z} - \tilde{Z}_{\text{lim}} > 0) \}
\]

where \( \tilde{Z}_{\text{lim}} \) is the limit value of state vector. The physical sense of this sub-norm is that the exceedance probability of the control quantity is minimized subjected to arbitrary random inputs. See Fig. 3.

It is clear that other state vectors could be out of control if the control norm is designed dependently on only one state vector. Therefore, a penalty function \( P_A \) is introduced into the following norms if the displacement of system for safety is defined to be the control quantity, and the acceleration of system for comfort is considered here as the constraint.

(a) Mean sub-norm

\[
\min (J') = \min \{ E[\tilde{D}] + P_A \}
\]

\[
= \min \{ E[\tilde{D}] \}
\]

\[
= \min \{ E[\tilde{D}] + 10^4 \}
\]

const rai nt s at i sf i ed o t her w i se

(b) Mean-standard deviation sub-norm

\[
\min (J') = \min \{ E[\tilde{D}] + r \times \sigma[\tilde{D}] + P_A \}
\]

\[
= \min \{ E[\tilde{D}] + r \times \sigma[\tilde{D}] \}
\]

\[
= \min \{ E[\tilde{D}] + r \times \sigma[\tilde{D}] + 10^4 \}
\]

const rai nt s at i sf i ed o t her w i se

(c) Exceedance probability sub-norm

\[
\min (J') = \min \{ P(\tilde{D} - \tilde{D}_{\text{lim}} > 0) + P_A \}
\]

\[
= \min \{ P(\tilde{D} - \tilde{D}_{\text{lim}} > 0) \}
\]

\[
= \min \{ P(\tilde{D} - \tilde{D}_{\text{lim}} > 0) + 1.0 \}
\]

const rai nt s at i sf i ed o t her w i se
3.2 OPTIMIZATION NORM BASED ON ENERGY CONSUME

To a control system, the energy consume, sometimes is preferred that yields the forthcoming control sub-norms.

(a) Mean sub-norm
\[
\min(J') = \min\{E[U]\}\quad (17)
\]

(b) Mean-standard deviation sub-norm
\[
\min(J') = \min\{E[U]+r\times\sigma[U]\}\quad (18)
\]

(c) Exceedance probability sub-norm
\[
\min(J') = \min\{P(U - \tilde{U}_\text{lim} > 0)\}\quad (19)
\]

where \(\tilde{U}_\text{lim}\) is the limit value of control force vector.

3.3 OPTIMAL PERFORMANCE CONTROL NORM BASED ON STATE VECTOR AND ENERGY CONSUME

When the displacement of system is taken as the control quantity with the constraints of the acceleration and the control force, a penalty function \(PAF\) is considered here. The control sub-norms are

(a) Mean sub-norm
\[
\min(J') = \min\{E[\tilde{D}] + PAF\}
= \min\{E[\tilde{D}]\} \quad \text{constraints satisfied}
= \min\{E[\tilde{D}] + 10^{4}\} \quad \text{otherwise}
\]

(b) Mean-standard deviation sub-norm
\[
\min(J') = \min\{E[\tilde{D}] + r\times\sigma[\tilde{D}] + PAF\}
= \min\{E[\tilde{D}] + r\times\sigma[\tilde{D}]\} \quad \text{constraints satisfied}
= \min\{E[\tilde{D}] + r\times\sigma[\tilde{D}] + 10^{4}\} \quad \text{otherwise}
\]

(c) Exceedance probability sub-norm
\[
\min(J') = \min\{P(\tilde{D} - \tilde{D}_\text{lim} > 0) + PAF\}
= \min\{P(\tilde{D} - \tilde{D}_\text{lim} > 0)\} \quad \text{constraints satisfied}
= \min\{P(\tilde{D} - \tilde{D}_\text{lim} > 0) + 1.0\} \quad \text{otherwise}
\]

4. COMPARISON ON OPTIMAL PERFORMANCE CONTROL NORMS

Consider an actively controlled SDOF system (see Fig. 4) subjected to random ground motion as the case for the comparison of the above norms. The properties of the system are as follows: (1) the mass of the first floor is \(m = 1 \times 10^5\) kg; (2) The natural circular frequency of the uncontrolled structural system is \(\omega_0 = 11.22\) rad/sec; (3) the control force of the actuator is denoted by \(f(t)\); and (4) \(\alpha\) represents the inclination angle of the tendon with respect to the base. The damping ratio is assumed to be 0.05. The system is subjected to random ground motion simulated with peak acceleration of 0.11 \(g\) employing the stochastic ground motion model[17]. 221 representative points with corresponding assigned-probabilities using the tangent spheres method[11] and representative time histories of ground accelerations are yielded. Additionally, the limited inter-story drift, the limited inter-story velocity, the limited floor acceleration and the limited control force of the structure are assumed to be 10 mm, 100 mm/sec, 3000 mm/sec\(^2\) and 200 kN, respectively.

A toolkit function of Matlab, \texttt{fmincon} is used in the case, which turns to the sequence quadratic programming method (SQR method, for short) to resolve the sub-problem of quadratic programming in each iteration. It is a local optimization method based on Kuhn-Tucker equation.

The exemplified three performance control norms are acceleration constrained displacement controlled norm, input force controlled norm and acceleration and input force constrained displacement controlled norm, respectively, where the optimization results are shown in Tables 1 ~ 3.

One sees that the acceleration and input force constrained displacement controlled norm is more comprehensive than the acceleration constrained displacement controlled norm and the input force controlled norm. Secondly, the parameters
optimization values of the mean sub-norm and the mean standard deviation sub-norm are nearly the same in the three classes of control norms, the maximum difference being 3.78%. It is due to the first moment occupying the main part of probabilistic properties, revealing that the mean sub-norm can cover the mean-standard deviation sub-norm. Thirdly, the acceleration constrained displacement controlled norm and the acceleration and input force constrained displacement controlled norm involved in the optimal parameters values of the exceedance probability sub-norm are equivalent that indicates the input force constraint not effective in the exceedance probability sub-norm. But they are different involved in the mean sub-norm because the mean sub-norm pays greater effort on minimizing the first moment which strengthens the constraint of the control force. However, the exceedance probability sub-norm focusing on the reliability maximized has no more requirements to the control force. So it is more economic in structural performance controls.

Comparison on optimal control policies of the three classes of norms is shown in Table 4 based on stochastic optimal control of the SDOF system subjected to random ground motion. It reveals that the effectiveness of responses control hinges on the physical meanings of the optimal control norms, i.e., the input force controlled norm underlies the control force minimized in the probabilistic sense, and may lead to amplifying certain responses instead of reducing the responses that, of course, could be improved by customizing the corresponding constraints. Compared to other control norms, the acceleration and input force constrained displacement controlled norm is more comprehensive. Among them the sub-norms exceedance probability sub-norm is the most reasonable. It means that the multi-penalty norm in the single-objective performance norms behaves more comprehensively than the single-penalty norm and non-penalty norm which is the prime norm to the structural single-objective control.

It is worth noting that the above optimizations are mainly involving the controlled displacement of system constrained by the acceleration and energy consumes. It is also feasible to relocate the controlled objective and its constraints.

5. CONCLUSIONS

A family of optimal performance control policies of structural stochastic optimal controls are proposed based on the probability density evolution theory and numerical optimizing strategies. These optimal performance control norms includes the acceleration constrained displacement controlled norm, the input force controlled norm and the acceleration and input force constrained displacement controlled norm in term of the physical requirements of the interested quantities, to each of which three sub-norms are discussed that are step-up and gradually subtle from the mean sub-norm to the exceedance probability sub-norm. A linear single-degree-of-freedom system subjected to random ground motion is investigated for the comparative purpose. The results show that the effectiveness of responses control hinges on the physical meanings of the optimal control norms.

ACKNOWLEDGEMENTS

The support of the Natural Science Foundation of China for Innovative Research Groups (Grant No. 50621062) is gratefully appreciated. Appreciation also goes to Mr. Wenliang Fan for his valuable advices in the Dirac-sequence approach to dynamic reliability assessment.
Figure 1 Physical sense of mean sub-norm.       Figure 2 Physical sense of mean-standard deviation sub-norm.

Figure 3 Physical sense of exceedance probability sub-norm.      Figure 4 Earthquake-excited SDOF structure with active tendon control system.

Table 1 The optimization results of acceleration constrained displacement controlled norm  
(Constrained acc. value: 3000 mm/sec², limited dis. value: 10 mm )

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Mean sub-norm</th>
<th>Mean-std. d sub-norm</th>
<th>Exceedance prob. sub-norm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Q_0$</td>
<td>$Q_1$</td>
<td>$r$</td>
</tr>
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<td>Initial value</td>
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<td>100</td>
<td>$10^{-10}$</td>
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<tr>
<td>Opt. value</td>
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<td>13983.4</td>
<td>$10^{-10}$</td>
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<tr>
<td>Obj. value</td>
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<td>1.06 mm</td>
<td>$10^{-10}$</td>
</tr>
<tr>
<td>Ite. number</td>
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<td>27</td>
<td>23</td>
</tr>
</tbody>
</table>

Table 2 The optimization results of input force controlled norm(Limited force value: 10 kN.)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Mean sub-norm</th>
<th>Mean-std. d sub-norm</th>
<th>Exceedance prob. sub-norm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Q_0$</td>
<td>$Q_1$</td>
<td>$r$</td>
</tr>
<tr>
<td>Initial value</td>
<td>100</td>
<td>100</td>
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<tr>
<td>Opt. value</td>
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<td>$10^{-8}$</td>
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</table>

Table 3 The optimization results of acceleration- and input force constrained displacement controlled norm(Constrained acc. value: 3000 mm/sec², constrained force value: 200 kN, limited dis. value: 10 mm)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Mean sub-norm</th>
<th>Mean-std. d sub-norm</th>
<th>Exceedance prob. sub-norm</th>
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<td>$Q_1$</td>
<td>$r$</td>
</tr>
<tr>
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<tr>
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<td>23</td>
</tr>
</tbody>
</table>

Table 4 Comparison on optimal control policies of three classes of norms(*Efficiency is defined as (Unc.-Con.)/Unc.)

<table>
<thead>
<tr>
<th>Control policies</th>
<th>Acc. constrained displacement controlled</th>
<th>Input force controlled</th>
<th>Acc.- and input force constrained displacement controlled</th>
</tr>
</thead>
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### Extreme values of displacements (mm)

<table>
<thead>
<tr>
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<th>Mean</th>
<th>Exc. prob.</th>
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<th>Exc. prob.</th>
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### Extreme values of accelerations (mm/sec²)

<table>
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<th>Mean</th>
<th>Exc. prob.</th>
<th>Mean</th>
<th>Exc. prob.</th>
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<td>3602.66</td>
<td>3602.66</td>
<td>3602.66</td>
<td>3602.66</td>
<td>3602.66</td>
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<tr>
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<tr>
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### Extreme values of control forces (kN)

<table>
<thead>
<tr>
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<th>Mean</th>
<th>Exc. prob.</th>
<th>Mean</th>
<th>Exc. prob.</th>
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<td>1745.59</td>
<td>1745.59</td>
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<td>1745.59</td>
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<tr>
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**REFERENCES**
